Please note there is a typo at the end of the video.
In the last matrix, the entry in the 3rd row, 4th column should be a -1.
The final solution is x1 = 4, x2 = -3 and x3 = -1.

Elementary Row Operations Part 2

> *Theorem 1:* Two $m \times n$ linear systems with corresponding augmented matrices that are *row equivalent* have *exactly the same set of solutions.*

Example 7: For the given linear system use elementary row operations to reduce the corresponding augmented matrix to row echelon form. Use theorem 1 and *back-substitution* to solve the linear system.

$$\begin{cases} x_1 + 2x_2 - x_3 = -1 \\ 3x_1 + 5x_2 - 2x_3 = -1 \\ 2x_1 + 2x_2 + x_3 = 1 \end{cases} \tag{1}$$

make 0's

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 3 & 5 & -2 & -1 \\ 2 & 2 & 1 & 1 \end{array}\right] \underset{\substack{R_3 := R_3 - 2R_1 \\ R_2 := R_2 - 3R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & \boxed{-2} & 3 & 3 \end{array}\right] \underset{R_2 := -R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & -2 & 3 & 3 \end{array}\right]$$

make 0

$$\underset{R_3 := R_3 + 2R_2}{\sim} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right] \end{array} \equiv \begin{cases} x_1 + 2x_2 - x_3 = -1 \\ \quad\ \ x_2 - x_3 = -2 \\ \qquad\quad\ x_3 = 5 \end{cases}$$

$$x_2 - 5 = -2 \qquad\qquad x_1 + 2(3) - 5 = -1$$
$$x_2 = 3 \qquad\qquad x_1 + 1 = -1$$
$$x_1 = -2$$

$$x_1 = -2, \ x_2 = 3, \ x_3 = 5$$